

# Finite CONV\* Banks for Joint Warped-Pixel Transfer

Addendum to Compact Ordered Nodal Variation

## I. FINITE CONV\* BANKS FOR JOINT WARPED-PIXEL TRANSFER

Let  $Y : \Lambda \rightarrow \mathbb{R}^c$  be a finite Cartesian source field. CONV assigns signed currents to source intervals, synthesizes two Cartesian factor orders, and combines them by a scalar action coordinate. Joint current admission then produces a continuous tensor-quintic control lattice  $P(Y)$ . This section compiles that construction and its warped-pixel functionals into finite banks. The coefficients carry the admitted source state, the continuous synthesis phase, and the integration geometry.

### A. Finite global action banks

Let  $\eta_a \in [0, 1]$  be the nodal order coordinate at source site  $a$  and  $d_M(x, a)$  the distance of the source-conditioned positive-definite metric. At an off-node query  $x$ , define

$$h_M(x, a) = d_M(x, a)^{-2}, \quad \beta(x) = \frac{\sum_{a \in \Lambda} h_M(x, a) \eta_a}{\sum_{a \in \Lambda} h_M(x, a)}. \quad (1)$$

The value at a source node is assigned by cardinal extension,  $\beta(a) = \eta_a$ . For a finite query set, the two sums are applications of one finite matrix to the columns  $\eta$  and  $\mathbf{1}$ . Positivity gives  $\min_a \eta_a \leq \beta(x) \leq \max_a \eta_a$ .

**Proposition 1** (Global FIR realization). *Suppose  $M$  is constant and source coordinates are in lattice units. Let  $m$  be the indicator of  $\Lambda$ , extended by zero, and let  $\phi \in [0, 1]^2$  be an off-node phase. On the finite set of required source and query differences, put*

$$k_\phi[r] = ((r + \phi)^T M (r + \phi))^{-1}. \quad (2)$$

With  $(k * f)[n] = \sum_a k[n - a] f[a]$ , the global action satisfies

$$\beta(n + \phi) = \frac{(k_\phi * (m\eta))[n]}{(k_\phi * m)[n]}. \quad (3)$$

*Proof.* For a constant metric,  $d_M(n + \phi, a)^2 = (n - a + \phi)^T M (n - a + \phi)$ . Substitution into (1) gives the numerator and denominator in (3). Zero extension of  $m$  includes precisely the finite source domain. The node value follows from the dominant self term as  $x \rightarrow a$ .  $\square$

Each phase therefore determines two finite convolutions with a shared kernel. Zero padding to the full linear-convolution extent permits their Fourier execution. For a spatially varying metric, the corresponding bank is  $h_M(n + \phi, a)$  with both indices retained. Metric construction and distance evaluation establish this bank once for a fixed source and query geometry. The bilinear nodal coordinate

$\beta^* = I_h \eta$  supplies a separate compact choice. For either coordinate the factor bank below has the same form, and changing the coordinate gives the identity

$$\mathcal{T}_{\beta^*} Y - \mathcal{T}_\beta Y = (\beta^* - \beta)(\mathcal{T}_{yx} Y - \mathcal{T}_{xy} Y). \quad (4)$$

### B. Active-face synthesis and two-dimensional composition

On a unit source interval, write  $a_k = K_k Y$  for the five raw currents,  $\delta = DY$  for the source difference, and  $eY$  for the left endpoint. The endpoint closures are included in  $K_k, D, e$ . The integrated Bernstein tails are

$$\tau_k(u) = \sum_{j=k+1}^5 B_j^5(u), \quad 0 \leq k \leq 4. \quad (5)$$

Let  $J$  be the support of the admitted current. On a nonempty active face, the Euclidean signed-fibre projection gives

$$c_k = \begin{cases} a_k + \frac{\delta - \sum_{l \in J} a_l}{|J|}, & k \in J, \\ 0, & k \notin J. \end{cases} \quad (6)$$

The ordered ledger and projection determine  $J$  from the input line.

**Proposition 2** (Face-indexed FIR synthesis). *For  $J \neq \emptyset$ , define*

$$\begin{aligned} \bar{\tau}_J(u) &= |J|^{-1} \sum_{k \in J} \tau_k(u), \\ W_J(u) &= e + \bar{\tau}_J(u) D \\ &\quad + \sum_{k \in J} (\tau_k(u) - \bar{\tau}_J(u)) K_k. \end{aligned} \quad (7)$$

*Then the admitted factor value equals  $W_J(u)Y$ . For the empty face,  $W_\emptyset(u) = e$ .*

*Proof.* Insert (6) into  $eY + \sum_k \tau_k(u) c_k$  and collect the coefficients of  $\delta$  and  $a_k$ . If  $J$  is empty, all admitted currents vanish and synthesis returns the anchor.  $\square$

There are 31 nonempty face families. Each is polynomial in  $u$  of degree at most five and has six-tap interior support. Lineage is retained in the choice of family. The factor can consequently be evaluated by one stencil after its face has been established.

For a source cell  $(i, j)$  and phase  $(u, v)$ , let  $W_{i,j+s}^x(u; r)$  be the first-factor stencil on row  $j + s$ . Let  $W_{i,j,u}^y(v; s)$  be the second-factor stencil established from the line of first-factor values at  $i + u$ .

**Theorem 1** (Conditioned two-dimensional factor bank). *The horizontal-vertical order has the finite representation*

$$K_{i,j,u,v}^{xy}[r,s] = W_{i,j,u}^y(v;s)W_{i,j+s}^x(u;r),$$

$$(\mathcal{T}_{xy}Y)(i+u, j+v) = \sum_{r,s} K_{i,j,u,v}^{xy}[r,s]Y_{i+r,j+s}. \quad (8)$$

*Constructing  $K^{yx}$  with exchanged axes gives the complete blended bank*

$$K_B = (1 - \beta)K^{xy} + \beta K^{yx}. \quad (9)$$

*Proof.* Apply (7) first to each source row and then to the resulting transverse line. Substitution of the first finite sum into the second gives (8). The convex factor blend gives (9).  $\square$

The interior support is six by six. Different source rows may have different first-factor faces, so the bank is generally nonseparable. On an interval of  $u$  where the second-factor face is fixed, its coefficients are independent of  $u$ ; the first-factor values remain quintic in  $u$ . Each order is then bicubic on the corresponding phase region. With bilinear  $\beta^*$ , the blend has bidegree at most (6,6) on a region where both orders have fixed faces. Ledger-choice and active-face inequalities certify such regions; phase evaluation within a certified region reuses the same polynomial coefficients.

For an endpoint-aligned rational lattice, write the reduced source-coordinate step on each axis as  $a_d/b_d$ . The fractional phases belong to the  $b_d$  residue classes modulo one. Their Cartesian product supplies at most  $b_x b_y$  phase pairs, with boundary closures and admitted face indices retained as bank indices. Continuous query coordinates evaluate the same polynomial families directly.

### C. Compilation through joint current admission

The joint atlas uses a fixed fifth-step collocation map followed by shared range and current admission. Let  $B(Y)$  be the degree-elevated bilinear baseline,  $\tilde{P}(Y)$  the collocation proposal, and  $\hat{P}(Y)$  its range projection. Assign each nonvertex control to one shared entity  $g$ . If  $\alpha_g$  is its simultaneous capacity and  $\tau$  the common completion parameter, define

$$\lambda_g = \alpha_g + \tau(1 - \alpha_g), \quad P_e = B_e + \lambda_g(\hat{P}_e - B_e), \quad e \in g. \quad (10)$$

Vertex controls retain their source values.

**Theorem 2** (Source-state factorization). *Let  $S(Y)$  record the faces and order coordinates used at collocation nodes, the selected range-bound source indices, and the admitted values of  $\lambda_g$ . For each channel there is a finite matrix  $A_{S(Y)}$  such that*

$$P(Y) = A_{S(Y)}Y. \quad (11)$$

*Proof.* With the recorded faces and coordinates, (9) makes each collocation value a linear source stencil. Inverse Bernstein collocation and bilinear degree elevation are fixed linear maps. Range projection selects one of three linear values: the proposal, the selected lower-bound sample, or

the selected upper-bound sample. Equation (10) then combines two linear stencils with recorded scalar coefficients. Assembling the shared entities gives  $A_{S(Y)}$ .  $\square$

For fixed  $Y$ , this factorization retains the exact admitted controls and therefore their cardinality, continuity, support-range bounds, and current cones. State discovery consists of line scans, finite projections, support extrema, incident-constraint minima, and the common ray reduction. Once that state is available, a source stencil can be propagated through every linear stage or the intermediate control lattice can be retained. Both evaluate (11).

### D. Point and area banks on the admitted atlas

Use source-pixel coordinates on the source rectangle  $\Omega$ . Let  $F$  map target coordinates to source coordinates and let  $G = F^{-1}$ . Assume the map is nonsingular and has no denominator zero on the domain of integration. Put  $Q = G(\Omega)$  and extend the source field by zero outside  $Q$  in target coordinates. On a source cell  $C$ , write

$$U_C(u, v) = \sum_{i,j=0}^5 P_{C,ij} B_i^5(u) B_j^5(v). \quad (12)$$

Point synthesis uses the continuous phase bank

$$\Psi_{u,v}[i, j] = B_i^5(u) B_j^5(v). \quad (13)$$

For a target basin  $E$  of positive area, let  $S_C = F(E \cap Q) \cap C$  and  $J_G = |\det DG|$ . Define the cell moment bank

$$M_{C,ij}(E, G) = \int_{S_C} B_i^5(u) B_j^5(v) J_G(u, v) du dv. \quad (14)$$

A change of variables gives

$$(\mathcal{B}_F Y)_E = \frac{1}{|E|} \sum_{C,ij} M_{C,ij} P_{C,ij}. \quad (15)$$

After assembly of shared controls and division by  $|E|$ , denote this bank by  $\mathbf{M}_E$ . The complete source-to-output operators are

$$(\mathcal{W}_F Y)(q) = \Psi_{F(q)}^T A_{S(Y)} Y, \quad (\mathcal{B}_F Y)_E = \mathbf{M}_E^T A_{S(Y)} Y. \quad (16)$$

**Proposition 3** (Positivity and conservation). *The exact moment weights are nonnegative and satisfy*

$$\sum_{C,ij} M_{C,ij} = |E \cap Q|. \quad (17)$$

For any basin partition  $\{E_b\}$ ,

$$\sum_b |E_b| (\mathcal{B}_F Y)_{E_b} = \int_{(\cup_b E_b) \cap Q} U_Y(F(q)) dq. \quad (18)$$

*Proof.* The Bernstein basis and  $J_G$  are nonnegative. Summing the basis gives one on each source cell. The cell footprints partition  $F(E \cap Q)$  up to boundaries of zero area, and change of variables gives  $|E \cap Q|$ . Summation over the target partition proves the second identity.  $\square$

For numerical moment construction, a constant reference  $R$  can be integrated through target coverage:

$$(\mathcal{B}_F Y)_E = \frac{R|E \cap Q|}{|E|} + \frac{1}{|E|} \sum_{C,i,j} (P_{C,ij} - R) M_{C,ij}. \quad (19)$$

This identity holds for any  $R$ . A reference chosen by a fixed linear functional of the controls preserves the assembled linear-bank form when approximate moments are substituted.

#### E. Exact affine moment banks

For a rectangular footprint  $[a, b] \times [c, d]$  in a cell and constant density  $\rho$ , define

$$A_i(t) = \frac{1}{6} \sum_{j=i+1}^6 B_j^6(t), \quad 0 \leq i \leq 5. \quad (20)$$

The Bernstein derivative identity telescopes to  $A'_i = B_i^5$ , yielding

$$M_{C,ij} = \rho[A_i(b) - A_i(a)][A_j(d) - A_j(c)]. \quad (21)$$

For a general affine map the cell footprint is polygonal and the integrand is polynomial of total degree at most ten. Its monomial moments can be obtained by an oriented boundary sum:

$$\int_D x^a y^b dx dy = \frac{1}{a+1} \oint_{\partial D} x^{a+1} y^b dy. \quad (22)$$

On each line segment, affine parameterization reduces the integrand to a polynomial in one variable. Equations (21) and (22) thus determine finite exact affine banks directly from footprint vertices.

#### F. Exact projective moment banks

Write  $G$  in homogeneous coordinates with nonsingular matrix  $H_G$  and affine denominator  $\ell(x, y)$ . Its area density is

$$J_G(x, y) = \frac{|\det H_G|}{|\ell(x, y)|^3}. \quad (23)$$

Choose a nonzero scale  $c_0$  of the same sign as  $\ell$  on the footprint, and write  $\ell = c_0 t$ , where  $t = \gamma + \alpha x + \beta y > 0$ . Let  $\rho = |\det H_G|/|c_0|^3$ .

**Theorem 3** (Finite projective edge compilation). *Every entry of (14) on a polygonal cell footprint is a finite expression in its vertices, the map coefficients, rational operations, and logarithms of positive denominator values.*

*Proof.* For  $\alpha \neq 0$ , transform to  $(t, y)$  so that  $x = (t - \gamma - \beta y)/\alpha$  and  $dx dy = |\alpha|^{-1} dt dy$ . The transformed polygon  $D$  is given counterclockwise orientation. Expand one tensor Bernstein numerator after substitution as  $\sum_{k,l} b_{kl} t^k y^l$ , with  $k + l \leq 10$ . Its bank entry is

$$M = \frac{\rho}{|\alpha|} \sum_{k,l} b_{kl} I_{kl}(D), \quad I_{kl}(D) = \int_D t^{k-3} y^l dt dy. \quad (24)$$

Green's theorem gives

$$I_{kl}(D) = -\frac{1}{l+1} \oint_{\partial D} t^{k-3} y^{l+1} dt. \quad (25)$$

For an oriented edge from  $(t_0, y_0)$  to  $(t_1, y_1)$  with  $t_0 \neq t_1$ , put  $a = (y_1 - y_0)/(t_1 - t_0)$  and  $b = y_0 - at_0$ . Its contribution is

$$-\frac{1}{l+1} \sum_{j=0}^{l+1} \binom{l+1}{j} a^j b^{l+1-j} \mathcal{L}_{k-3+j}(t_0, t_1), \quad (26)$$

where

$$\mathcal{L}_e(a, b) = \begin{cases} (b^{e+1} - a^{e+1})/(e+1), & e \neq -1, \\ \log(b/a), & e = -1. \end{cases} \quad (27)$$

An edge with  $t_0 = t_1$  contributes zero. If  $\alpha = 0$  and  $\beta \neq 0$ , exchange the source axes. If both vanish, the density is constant and (22) applies.  $\square$

The same edge moments serve all 36 basis numerators and every channel. Boundary terms on oppositely oriented shared edges cancel when their chosen Green primitives have the same edge trace. An oriented edge table can therefore reuse primitives across neighboring footprints while preserving each cell's polynomial coefficients.

For close positive  $a, b$ , set  $h = (b - a)/a$ . The primitive difference has the continuous form

$$\mathcal{L}_e(a, b) = \begin{cases} \frac{a^{e+1}}{e+1} \expm1((e+1) \log_1 p(h)), & e \neq -1, \\ \log_1 p(h), & e = -1. \end{cases} \quad (28)$$

This evaluates the power and logarithm differences without subtracting their nearly equal endpoint values. The factors  $a^j b^{l+1-j}$  in (26) still require control of cancellation for nearly constant-denominator edges. Scaled cell coordinates, selection of the larger of  $|\alpha|, |\beta|$ , and the geometry-selected positive quadrature in Section I-G provide evaluation across these configurations and the affine limit. The finite edge formula supplies an independent analytic reference for the geometric moment construction.

#### G. Geometric compilation of projective banks

Triangulate each clipped cell footprint. The canonical numerical construction uses a positive triangle rule selected by denominator geometry and evaluates  $J_G$  directly at its nodes. On a triangle  $T$ , let  $[\ell_-, \ell_+]$  be the same-sign denominator range and put

$$c_T = \frac{\ell_- + \ell_+}{2}, \quad z = \frac{\ell}{c_T} - 1, \quad r_T = \frac{\ell_+ - \ell_-}{2|c_T|} < 1. \quad (29)$$

The extrema occur at the three vertices. With  $\rho_T = |\det H_G|/|c_T|^3$ , the density is  $J_G = \rho_T(1 + z)^{-3}$ . The radius  $r_T$  is unchanged by nonzero rescaling of the homogeneous matrix.

Let  $\mathcal{Q}_K$  be a positive-weight triangle rule exact through total degree  $10+K$ , with physical weights  $w_l > 0$  satisfying  $\sum_l w_l = |T|$ . Define

$$\begin{aligned}\widehat{M}_{T,ij} &= \sum_{l=1}^{L_K} w_l J_G(p_l) B_i^5(p_{l,x}) B_j^5(p_{l,y}), \\ \widehat{I}_{T,a} &= \sum_{i,j} (P_{C,ij,a} - R_a) \widehat{M}_{T,ij}.\end{aligned}\quad (30)$$

Every channel uses the same 36 nonnegative moment weights. Forming these weights before channel contraction gives  $36L_K$  geometric contributions and  $36c$  control contributions per triangle.

For barycentric coordinates  $(1-u-v, u, v)$  on a triangle with vertices  $a, b, d$ , the node denominator is

$$\ell(p) = \ell(a) + u[\ell(b) - \ell(a)] + v[\ell(d) - \ell(a)]. \quad (31)$$

The three values used to select the rule therefore also evaluate its density. When  $r_T = 0$ , one constant density serves all nodes.

1) *A fixed geometric rule table:* Use the positive interior Xiao–Gimbutas triangle rules of degrees ten through sixteen. The following table fixes the admissible radii and node counts:

| $K$ | Maximum $r_T$ | Exact degree | $L_K$ |
|-----|---------------|--------------|-------|
| 0   | 0             | 10           | 25    |
| 1   | 0.000009      | 11           | 28    |
| 2   | 0.00036       | 12           | 33    |
| 3   | 0.0023        | 13           | 37    |
| 4   | 0.0074        | 14           | 42    |
| 5   | 0.0159        | 15           | 49    |
| 6   | 0.0276        | 16           | 55    |

Choose the first row containing  $r_T$ . If no row contains it, join the three edge midpoints and apply the same table to the four child triangles. The source controls and channel values do not enter this choice. The published rule coordinates and weights are retained from the Xiao–Gimbutas triangle tables.

**Proposition 4** (Finite geometric partition). *For a fixed nonsingular homography whose denominator has no zero on the footprint, midpoint subdivision terminates after finitely many levels.*

*Proof.* Compactness gives  $m_\ell = \min |\ell| > 0$ . On a triangle of diameter  $h$ ,  $\ell_+ - \ell_- \leq \|\nabla \ell\| h$  and  $|c_T| \geq m_\ell$ . Each midpoint child has half its parent’s diameter. After  $n$  levels,

$$r_T \leq \frac{\|\nabla \ell\| h_0}{2^{n+1} m_\ell}. \quad (32)$$

This eventually lies below the final table radius. Constant denominators use the first row directly.  $\square$

2) *Uniform accuracy of the moment operator:* The table is established from polynomial exactness and a uniform remainder. Write

$$S_K(z) = \sum_{k=0}^K (-1)^k \binom{k+2}{2} z^k. \quad (33)$$

For  $n \geq 1$ , the absolute series tail is

$$\begin{aligned}T_n(r) &= \sum_{k=n}^{\infty} \binom{k+2}{2} r^k \\ &= \frac{r^n}{2} \left[ \frac{r(1+r)}{(1-r)^3} + \frac{(2n+3)r}{(1-r)^2} + \frac{(n+1)(n+2)}{1-r} \right].\end{aligned}\quad (34)$$

The closed form follows by writing  $k = n+j$  and summing  $1, j, j^2$  against  $r^j$ . Hence  $|(1+z)^{-3} - S_K(z)| \leq T_{K+1}(r_T)$ .

**Theorem 4** (Geometry-only moment accuracy). *Let  $m_T = \int_T J_G$ . A positive rule exact through degree  $10+K$  satisfies*

$$\frac{\sum_{i,j} |\widehat{M}_{T,ij} - M_{T,ij}|}{m_T} \leq 2(1+r_T)^3 T_{K+1}(r_T). \quad (35)$$

*Every row of the geometric table makes the right-hand side at most  $\varepsilon_g = 10^{-9}$ . The same mass-relative bound holds after assembling all triangle pieces of a footprint.*

*Proof.* Put  $b_{ij} = B_i^5 B_j^5$ . Its total degree is at most ten, so  $\mathcal{Q}_K$  integrates  $b_{ij} S_K(z)$  exactly. Only the remainder  $R_K = (1+z)^{-3} - S_K(z)$  contributes to the moment defect. Positivity of the rule and  $\sum_{i,j} b_{ij} = 1$  give

$$\begin{aligned}\sum_{i,j} |\widehat{M}_{T,ij} - M_{T,ij}| &\leq \rho_T \left( \int_T |R_K| + \mathcal{Q}_K(|R_K|) \right) \\ &\leq 2|T| \rho_T T_{K+1}(r_T).\end{aligned}\quad (36)$$

Since  $m_T \geq |T| \rho_T (1+r_T)^{-3}$ , (35) follows. The right-hand side is nondecreasing in  $r_T$ ; substitution of each table endpoint gives the stated budget. The triangle inequality and additivity of Jacobian mass preserve it under assembly.  $\square$

The polynomial  $S_K$  supplies the proof of the fixed table. The numerical bank in (30) evaluates the full rational density. Runtime planning consists of the vertex range, table lookup and geometric subdivision.

**Proposition 5** (Pixel-functional accuracy). *For a target basin  $E$ , let  $D_{E,a} = \max_{C,ij} |P_{C,ij,a} - R_a|$  over its contributing cells. Substitution of the geometric moments into (19) satisfies*

$$|(\widehat{\mathcal{B}}_F Y)_{E,a} - (\mathcal{B}_F Y)_{E,a}| \leq \varepsilon_g D_{E,a} \frac{|E \cap Q|}{|E|}. \quad (37)$$

*Proof.* Contract the assembled absolute moment defect with the largest residual control and use  $\sum_T m_T = |E \cap Q|$ . Division by  $|E|$  gives the result.  $\square$

A constant field is reproduced through exact target coverage by choosing its constant value as  $R$ . For a partition of target basins, the absolute error in the area-weighted total is at most  $\varepsilon_g \sum_E D_{E,a} |E \cap Q|$ . A reference obtained from a fixed linear evaluation of the controls retains the source-state bank factorization. The geometric partition and its moments can be reused for any controls on the same lattice and warp.

3) *Verification of the fixed construction:* The degree-ten through degree-sixteen rules reproduce all reference-triangle monomials through their declared degrees with maximum relative discrepancy below  $3 \times 10^{-14}$  in the retained tables. Their nodes lie in the triangle and their weights are positive. Independent finite edge evaluation at 220 decimal digits checks all 36 computed moments for 57 triangle and denominator configurations. These include both sides of every positive radius threshold, skew and thin triangles, and denominator ratios through 1999 : 1. The largest measured mass-relative sum of absolute weight errors is  $8.70 \times 10^{-10}$ , including the floating-point subdivision and accumulation in the tested implementation.

A separate pixel-functional check uses six source families, six homographies and three output sizes. The families include smooth oscillations, alternating samples, noise, a constant field, an isolated impulse and nearly transparent samples. Against a reference evaluated with a  $10^{-12}$  pixel tolerance, 8,640 sampled comparisons have maximum premultiplied-channel difference  $1.33 \times 10^{-13}$ . All source families produce the same geometric rule and subdivision counts for a common map and lattice. The retained moment and pixel receipts accompany the direct-warped geometric compilation study.

#### H. Composition, reuse, and numerical enclosures

The complete dataflow is

$$Y \mapsto S(Y) \mapsto A_{S(Y)}Y \mapsto \{\Psi_q^T P, M_E^T P\}. \quad (38)$$

Independent lines within each factor share one raw-current bank. The next factor consumes their synthesized values. Shared-entity capacities are minima over incident constraints; the completion parameter is a global minimum. These associative reductions admit parallel trees, while exact lineage uses a prefix scan. Once  $P$  is fixed, target basins and channel contractions can be partitioned independently. Repeated phase pairs reuse synthesis weights; repeated footprints reuse moment banks; common polygon edges reuse oriented primitives. For  $N_T$  triangle pieces, moment-first quadrature has contraction work proportional to  $\sum_{T=1}^{N_T} (36L_T + 36c)$ , in addition to geometry construction.

The identities above hold in real arithmetic. A numerical enclosure can be assembled from storage, moment, and contraction errors. If stored controls satisfy  $|\hat{P}_e - P_e| \leq \delta_e$  and computed unnormalized exact-bank approximations satisfy  $|\hat{M}_e - M_e| \leq \mu_e$ , then

$$\left| \sum_e \hat{M}_e \hat{P}_e - \sum_e M_e P_e \right| \leq \sum_e |\hat{M}_e| \delta_e + \sum_e \mu_e |P_e|. \quad (39)$$

The final summation contributes its arithmetic enclosure and the result is divided by  $|E|$ . For reference-subtracted geometric banks, (37) supplies the moment-approximation contribution. Bounds on computed coverage, primitive evaluation, and summation are added at the corresponding

stages. Exact-bank positivity gives the sharper storage bound  $\max_e \delta_e |E \cap Q|/|E|$  when moments are exact.

Thus global admission, nonlinear factor state, joint control admission, and warped integration each have a finite compilation: a normalized global bank, a face-indexed two-dimensional stencil, a source-state matrix, and a geometry-dependent moment bank. Their composition retains the declared continuous source field and its conservative target functional.